Year 10

Surds

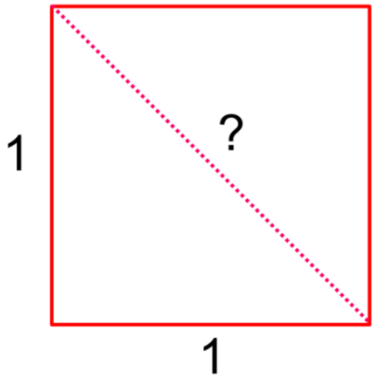
Name _____

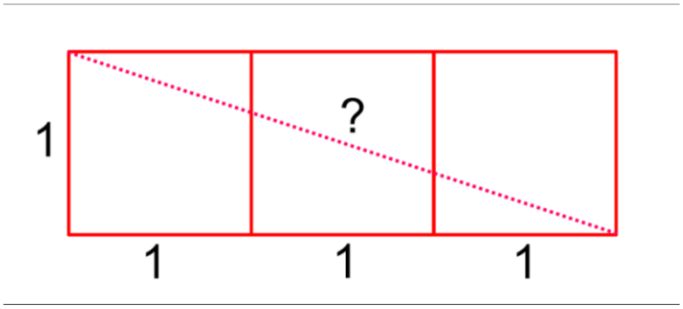
House _____

		Done	S.A
1	Introducing surds		
2	Manipulating irrational numbers		
3	Rationalising the denominator		

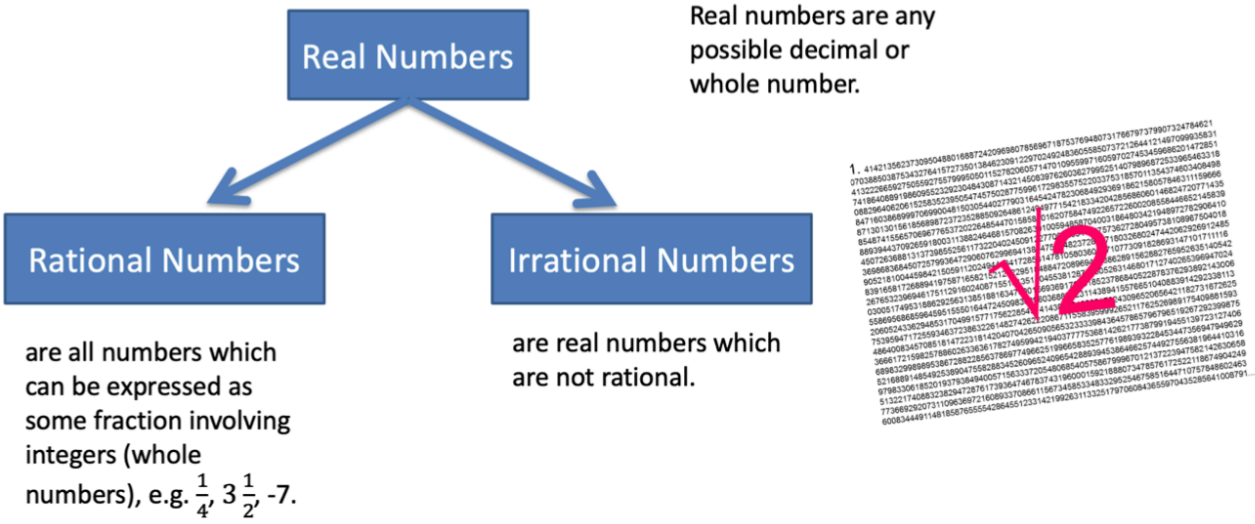
Surds (1) Introducing surds

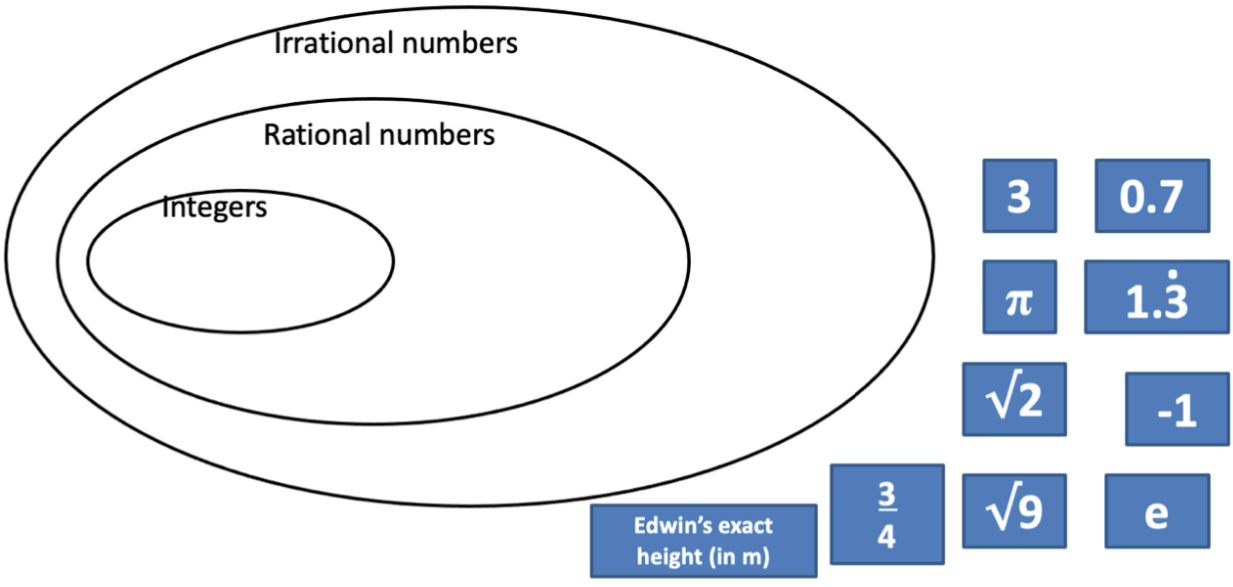
Do now: Calculate the question marks





Do now: Read the definitions below and sort the numbers into the correct sets


Real Numbers
Real numbers are any possible decimal or whole number.
Rational Numbers
Irrational Numbers
are all numbers which can be expressed as some fraction involving integers (whole numbers), e.g. $\frac{1}{4}$, $3\frac{1}{2}$, -7.
are real numbers which are not rational.
√2



A surd is a root of a number that cannot be simplified to a rational number.
Which of the following are surds?

$\sqrt{5^{-2}}$	Yes / No	$\sqrt{\frac{2}{8}}$	Yes / No
$\frac{1}{(\sqrt{5})^2}$	Yes / No	$\sqrt{\frac{2}{9}}$	Yes / No
$5^{\frac{1}{2}}$	Yes / No	$\sqrt{\frac{4}{9}}$	Yes / No
$4^{\frac{1}{2}}$	Yes / No	$\frac{2}{\sqrt{9}}$	Yes / No
$\frac{\sqrt{1}}{\sqrt{4}}$	Yes / No	$\frac{\sqrt{7}}{2}$	Yes / No
$\sqrt{\frac{1}{4}}$	Yes / No	$\sqrt{0.25}$	Yes / No
$\sqrt{1}$	Yes / No	$\sqrt{3}$	Yes / No
$\sqrt{4}$	Yes / No	$2\sqrt{3}$	Yes / No
$\sqrt{9}$	Yes / No	$3\sqrt{3}$	Yes / No
$\sqrt{36}$	Yes / No	$3\sqrt{4}$	Yes / No
$\sqrt{6}$	Yes / No	$\sqrt{5}$	Yes / No
$\sqrt{24}$	Yes / No	$\sqrt{5^2}$	Yes / No
$\sqrt{0.125}$	Yes / No	$\sqrt{0.01}$	Yes / No

Evaluate the following without a calculator:

$\sqrt{4} \times \sqrt{9} =$	$\frac{\sqrt{16}}{\sqrt{4}} =$
$\sqrt{25} \times \sqrt{4} =$	$\frac{\sqrt{100}}{\sqrt{25}} =$
$\sqrt{4} \times \sqrt{16} =$	$\frac{\sqrt{3600}}{\sqrt{100}} =$
$\sqrt{100} \times \sqrt{9} =$	$\frac{\sqrt{400}}{\sqrt{4}} =$
$\sqrt{36} \times \sqrt{100} =$	$\frac{\sqrt{81}}{\sqrt{9}} =$

What do you notice? Try the questions below

$\sqrt{2} \times \sqrt{32} =$	$\frac{\sqrt{50}}{\sqrt{2}} =$
$\sqrt{3} \times \sqrt{12} =$	$\frac{\sqrt{75}}{\sqrt{3}} =$
$\sqrt{a} \times \sqrt{b} =$	$\frac{\sqrt{a}}{\sqrt{b}} =$

Simplifying surds

$\sqrt{8} =$	$\sqrt{75} =$
$\sqrt{700} =$	$\sqrt{20} =$
$\sqrt{450} =$	$\sqrt{48} =$
$\sqrt{32} =$	$3\sqrt{200} =$
$2\sqrt{50} =$	$5\sqrt{45} =$
$4\sqrt{12} =$	

$\sqrt{3}\times\sqrt{5} =$	$6\times\sqrt{7} =$
$2\times\sqrt{3} =$	$\sqrt{5}\times\sqrt{6} =$
$\sqrt{5}\times 3 =$	$\sqrt{2}\times 2 =$
$\sqrt{2}\times\sqrt{8} =$	$5\sqrt{3}\times 4\sqrt{3} =$
$\sqrt{3}\times\sqrt{3} =$	$3\times 4\sqrt{3} =$
$2\sqrt{3}\times 2\sqrt{5} =$	$(3\sqrt{5})^2 =$
$3\sqrt{2}\times 3\sqrt{2} =$	$5\sqrt{8}\times 2\sqrt{2} =$
$\sqrt{18}\times 4\sqrt{2} =$	$2\sqrt{2}\times 3\sqrt{6} =$

1 Simplify the following:

a $\sqrt{8} =$

b $\sqrt{18} =$

c $\sqrt{50} =$

d $\sqrt{80} =$

e $\sqrt{72} =$

2 Simplify the following:

a $5\sqrt{80} =$

b $2\sqrt{125} =$

c $8\sqrt{12} =$

d $3\sqrt{72} =$

e $2\sqrt{28} =$

3 Simplify the following:

a $\sqrt{3}\times\sqrt{2}\times\sqrt{5} =$

b $\sqrt{27}\times\sqrt{3} =$

c $4\sqrt{3}\times 2 =$

d $5\times 2\sqrt{5} =$

e $2\sqrt{2}\times 2\sqrt{2} =$

f $7\sqrt{3}\times 2\sqrt{5} =$

g $6\sqrt{3}\times 2\sqrt{3} =$

4 Simplify the following:

a $\sqrt{8}\times 3\sqrt{2} =$

b $\sqrt{27}\times 2\sqrt{3} =$

c $3\sqrt{18}\times\sqrt{2} =$

d $2\sqrt{12}\times 3\sqrt{3} =$

5 Express the following as a single square root

(hint: do the steps of simplification backwards!)

a $3\sqrt{2} =$

b $2\sqrt{5} =$

c $5\sqrt{7} =$

d $4\sqrt{3} =$

6 Express the following as a single square root:

a $a\sqrt{b} =$

b $2\sqrt{k} =$

Simplify fully

$$\frac{\sqrt{12}}{\sqrt{48}}$$

Surds (2) Addition and subtraction

Do now: Decide whether each of the statements below are true or false

a. $\sqrt{9} + \sqrt{4} = \sqrt{13}$

b. $\sqrt{3} \times \sqrt{3} = 3$

c. $\sqrt{9} \times \sqrt{4} = \sqrt{36}$

d. $\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$

e. $\sqrt{200} = 10\sqrt{2}$

f. $\sqrt{200} = 2\sqrt{100}$

g. $\sqrt{200} = 2\sqrt{50}$

h. $\sqrt{2} = 1.4142136$

i. $\sqrt{2} = 1.414213562$

j. $\sqrt{2} = 1.4142$

k. $\sqrt{9} - \sqrt{4} = \sqrt{5}$

l. $\frac{\sqrt{36}}{\sqrt{4}} = \sqrt{9}$

m. $\sqrt{200} = 5\sqrt{8}$

o. $\sqrt{200} = \sqrt{20} \times \sqrt{10}$

Adding and subtracting surds – it's the same as collecting like terms

$\sqrt{3} + \sqrt{3} =$	$2\sqrt{5} + \sqrt{5} =$	$7\sqrt{7} + 7\sqrt{7} =$	$\sqrt{2} + \sqrt{8} =$	$3\sqrt{12} + \sqrt{27}$

Test your understanding

$\sqrt{3} + \sqrt{3} + \sqrt{3}$	$\sqrt{8} + \sqrt{18}$	$2\sqrt{5} + 2\sqrt{20}$	$3\sqrt{48} + \sqrt{12}$	$a\sqrt{b} + \sqrt{a^2 b}$

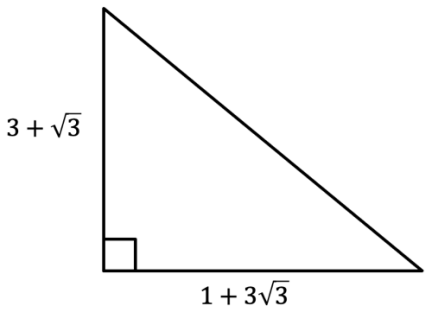
Examples – expanding and simplifying

$\sqrt{2}(3+\sqrt{2})$	$(\sqrt{8}+3)(\sqrt{2}+5)$	$(\sqrt{2}+1)(\sqrt{2}-1)$	$(\sqrt{5}-2)^2$

Test your understanding

$\sqrt{5}(2+\sqrt{3})$	$(1+\sqrt{3})(2+\sqrt{3})$	$(\sqrt{8}-1)(\sqrt{2}+3)$	$(3-2\sqrt{5})^2$

Extension – calculate the area of the triangle below



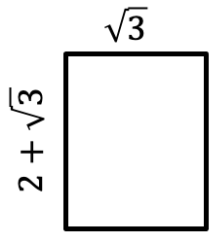
Exercise 2 – simplify the following without a calculator

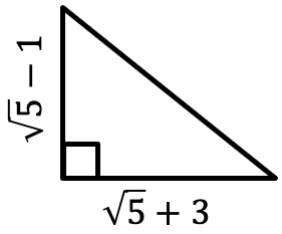
$\sqrt{8}+\sqrt{18}$	$\sqrt{12}-\sqrt{3}$	$\sqrt{20}+\sqrt{45}$	$\sqrt{3}+\sqrt{12}+\sqrt{27}$	$\sqrt{300}-\sqrt{48}$	$2\sqrt{50}+3\sqrt{32}$

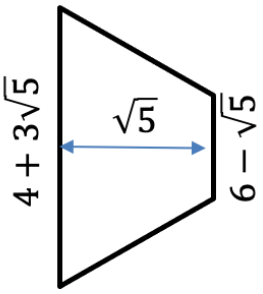
Simplify the following:
Expand and simplify the following, leaving your answers in the form $a + b\sqrt{c}$

$\sqrt{3}(2+\sqrt{3})$	$(\sqrt{3}+1)(2+\sqrt{3})$	$(\sqrt{5}-1)(2+\sqrt{5})$	$(\sqrt{7}+1)(2-2\sqrt{7})$	$(2-\sqrt{3})^2$

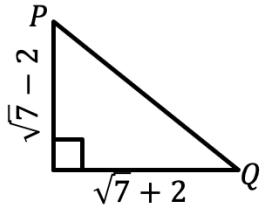
Determine the area of :







Find the length of PQ



Non calculator challenge

How many of the numbers

$3\sqrt{11}$ $4\sqrt{7}$ $5\sqrt{5}$ $6\sqrt{3}$ $7\sqrt{2}$

are greater than 10?

A 1 B 2 C 3 D 5

Surds (3) Rationalising the denominator

Do now:

1. *Which of the following are irrational?* $\sqrt{18}$, $\frac{1}{7}$, $\sqrt{3600}$, $-\frac{5}{4}$,π
2. $Simplify\ \sqrt{450}$
3. $8\sqrt{5} - \sqrt{5}$
4. $2\sqrt{5} \times 2\sqrt{5}$
5. $4\sqrt{5} \div 2\sqrt{5}$
6. $\sqrt{45} + \sqrt{125}$
7. $\sqrt{2}(2 + \sqrt{2})$
8. $What\ is\ the\ area\ of\ a\ square\ with\ side\ length\ of\ (\sqrt{2} + 1)$

Here's a surd. What could we multiply it by such that it's no longer an irrational number?

Worked example	Your turn
$\frac{3}{\sqrt{2}}$	$\frac{12}{\sqrt{3}}$
$\frac{1}{\sqrt{2}}$	$\frac{2}{\sqrt{6}}$
$\frac{6}{\sqrt{3}}$	$\frac{4\sqrt{2}}{\sqrt{8}}$
$\frac{8}{\sqrt{10}}$	$\frac{8}{\sqrt{2}}$
$\frac{10}{\sqrt{5}}$	$\frac{33}{\sqrt{11}}$
$\frac{7}{\sqrt{7}}$	$\frac{8\sqrt{2}}{\sqrt{8}}$

Extension

$$\frac{1}{\sqrt{2}+1}$$

$\frac{3}{\sqrt{6}-2}$	$\frac{2}{\sqrt{5}+2}$
$\frac{4}{\sqrt{3}+1}$	$\frac{7}{\sqrt{2}-7}$
$\frac{3}{2\sqrt{3}+1}$	$\frac{3}{\sqrt{7}+3}$

1. Rationalise the denominator and simplify the following:

$\frac{1}{\sqrt{5}+2} =$

$\frac{\sqrt{3}}{\sqrt{3}-1} =$

$\frac{\sqrt{5}+1}{\sqrt{5}-2} =$

$\frac{2\sqrt{3}-1}{3\sqrt{3}+4} =$

$\frac{5\sqrt{5}-2}{2\sqrt{5}-3} =$

2 Expand and simplify:

$(\sqrt{5}+3)(\sqrt{5}-2)(\sqrt{5}+1) =$

Rationalise the denominator, giving your answer in the form $a + b\sqrt{3}$.

$\frac{3\sqrt{3}+7}{3\sqrt{3}-5} =$

Solve $x(4-\sqrt{6}) = 10$ giving your answer in the form $a + b\sqrt{6}$.

Martin did this question.

Rationalise the denominator of $\frac{14}{2+\sqrt{3}}$

Here is how he answered the question.

$$\frac{14}{2+\sqrt{3}} = \frac{14\times(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{28-14\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}+3}$$

$$= \frac{28-14\sqrt{3}}{7}$$

$$= 4-2\sqrt{3}$$

Work out the mean of the three numbers $\sqrt{75}$, $\sqrt{75}$ and $\frac{6}{\sqrt{3}}$.

Give your answer in the form $b\sqrt{3}$ where b is an integer.

Solve $y(1+\sqrt{2})-\sqrt{2}=3$

Simplify

$$\frac{\sqrt{a+1}-\sqrt{a}}{\sqrt{a+1}+\sqrt{a}}$$

The diagram shows a solid cylinder.

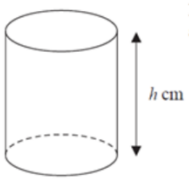


Diagram **NOT** accurately drawn

The cylinder has radius $4\sqrt{3}$ cm and height h cm.
The total surface area of the cylinder is $56\pi\sqrt{6}\ cm^2$.

Find the exact value of h.
Give your answer in the form $a\sqrt{2}+b\sqrt{3}$ where a and b are integers.
Show your working clearly.

A trapezium *ABCD* has an area of $5\sqrt{6}$ cm^2.

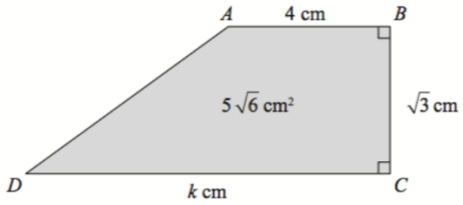


Diagram **NOT** accurately drawn

AB = 4 cm.
BC = $\sqrt{3}$ cm.
DC = k cm.

Calculate the value of k, giving your answer in the form $a\sqrt{b}-c$, where a, b and c are positive integers.

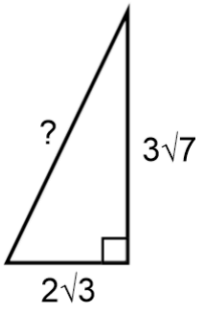
Using Surds

Increasingly Difficult Exercises

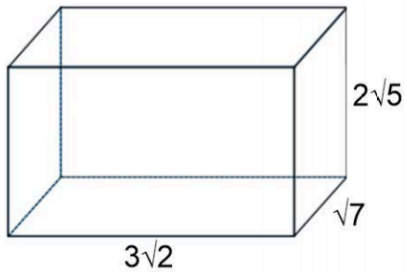
a)

Calculate the area of a rectangle measuring $4\sqrt{10}$ cm by $5\sqrt{2}$ cm.

b)



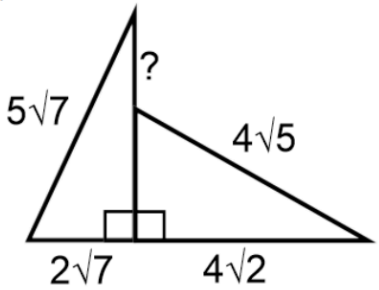
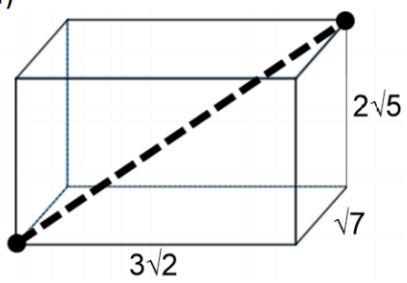
c) Volume?
Surface Area?



d)

The area of a triangle is $10\sqrt{5}$ cm². It is enlarged by a scale factor of $3\sqrt{2}$. What is the area of the enlarged triangle?

e)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